



ELECTRIC FIELD

Electric Field due to Point Charge



$$E = \frac{kq}{x^2}$$

Vector Form

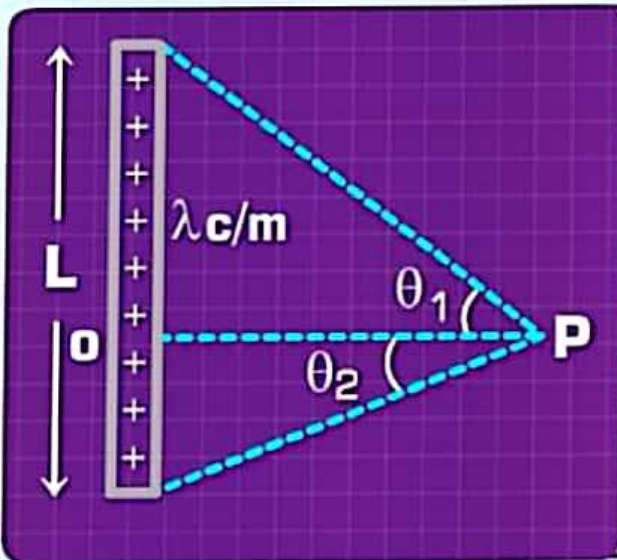
$$\vec{E} = \frac{kq}{x^3} \cdot \vec{x}$$

$$k = \frac{1}{4\pi\epsilon_0}$$

q = Charge ; x = Distance

If a charge q_0 is placed at a point in electric field, it experiences a net force \vec{F} on it, then electric field strength at that point can be $\vec{E} = \frac{\vec{F}}{q_0}$

ELECTRIC FIELD DUE TO A UNIFORMLY CHARGED ROD



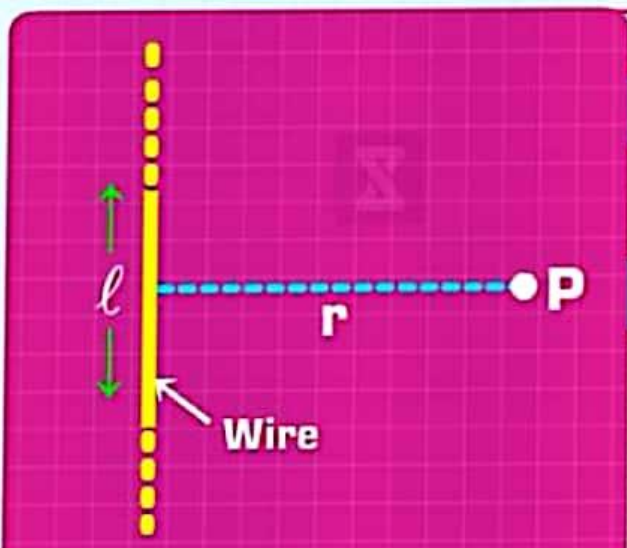
PARALLEL

$$E_{\parallel} = \frac{k\lambda}{r} (\cos\theta_2 - \cos\theta_1)$$

PERPENDICULAR

$$E_{\perp} = \frac{k\lambda}{r} (\sin\theta_2 - \sin\theta_1)$$

ELECTRIC FIELD DUE TO INFINITE WIRE ($\ell \gg r$)



Since $\ell \gg r \Rightarrow \theta_1 = \theta_2 = 90^\circ$

PERPENDICULAR

$$E_{\perp} = \frac{k\lambda}{r} (\sin 90^\circ + \sin 90^\circ) \Rightarrow E_{\perp} = \frac{2k\lambda}{r}$$

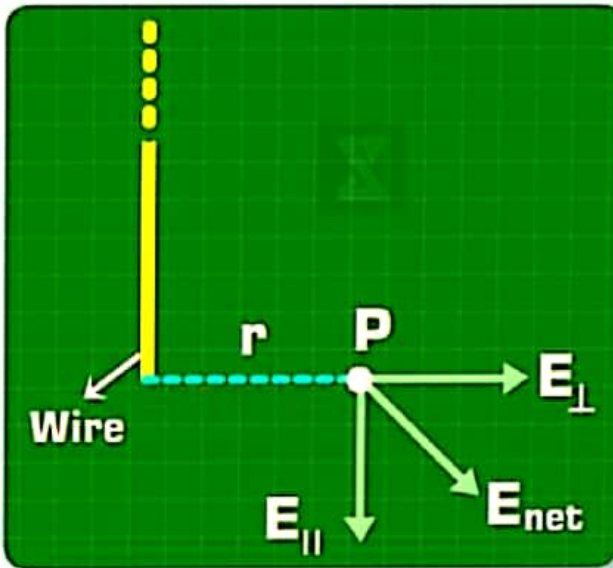
PARALLEL

$$E_{\parallel} = \frac{k\lambda}{r} (\cos 90^\circ - \cos 90^\circ) \Rightarrow E_{\parallel} = 0$$

At P, $E_{\text{net}} = E_{\perp} + E_{\parallel}$

$$E_{\text{net}} = \frac{2k\lambda}{r}$$

ELECTRIC FIELD DUE TO SEMI INFINITE WIRE



$$\theta_1 = 90^\circ, \quad \theta_2 = 0^\circ$$

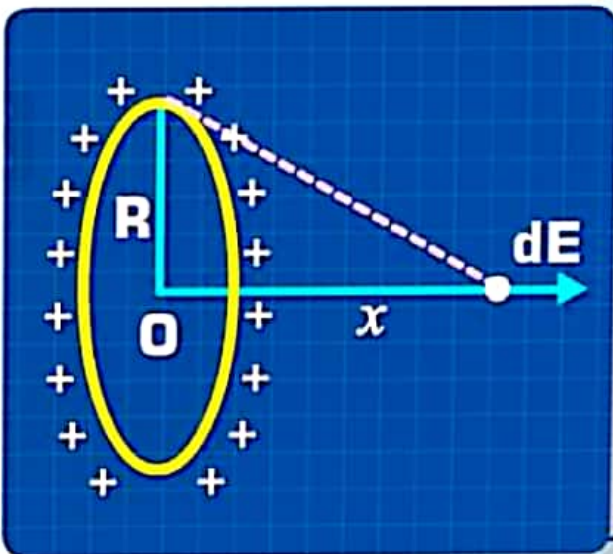
PERPENDICULAR

$$E_{\perp} = \frac{k\lambda}{r} (\sin 90^\circ + \sin 0^\circ) = \frac{k\lambda}{r}$$

PARALLEL

$$E_{\parallel} = \frac{k\lambda}{r} (\cos 0^\circ - \cos 90^\circ) = \frac{k\lambda}{r}$$

ELECTRIC FIELD DUE TO UNIFORMLY CHARGED RING

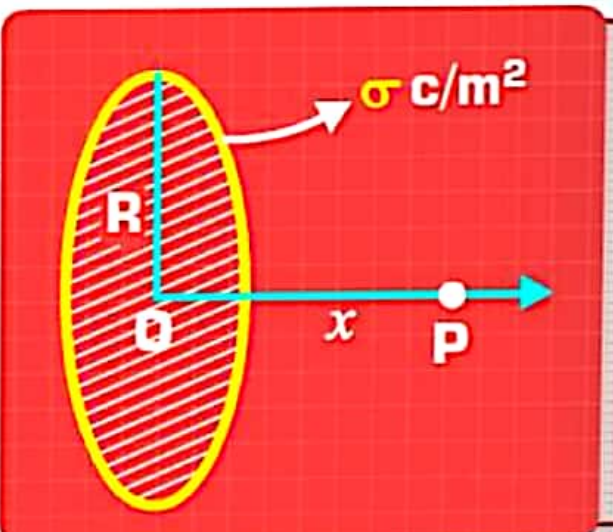


$$E = \frac{kQx}{(R^2 + x^2)^{3/2}}$$

For maxima, $x = \pm \frac{R}{\sqrt{2}}$

$$E_{\max} = \pm \frac{2}{3\sqrt{3}} \frac{kQ}{R^2}$$

ELECTRIC FIELD ON THE AXIS OF DISC



$$E = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{x}{\sqrt{x^2 + R^2}} \right] \text{ [along the axis]}$$

If $x \gg R$

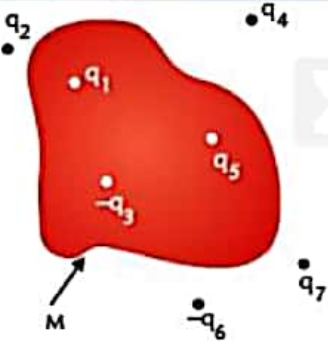
$$E = 0$$

If $x \ll R$

$$E = \frac{\sigma}{2\epsilon_0} (1 - 0) = \frac{\sigma}{2\epsilon_0}$$

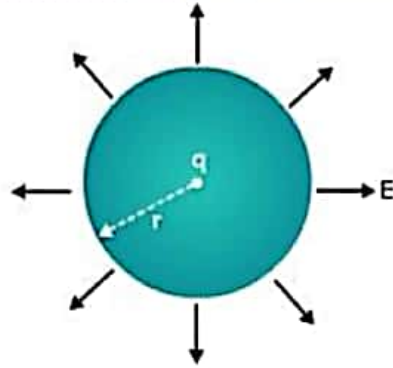
ELECTRIC FIELD STRENGTH

Gauss's Law



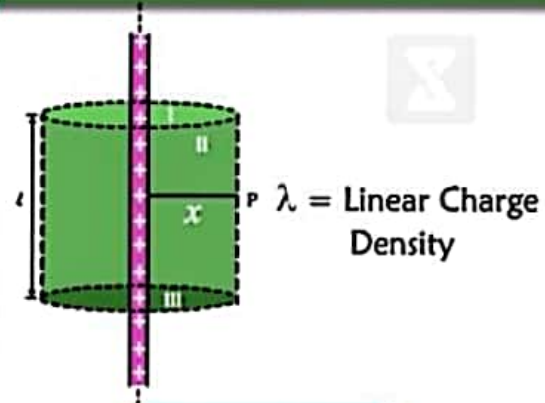
$$\oint_M \vec{E} \cdot d\vec{S} = \frac{q_1 + q_5 - q_3}{\epsilon_0}$$

Electric Field due to a Point Charge



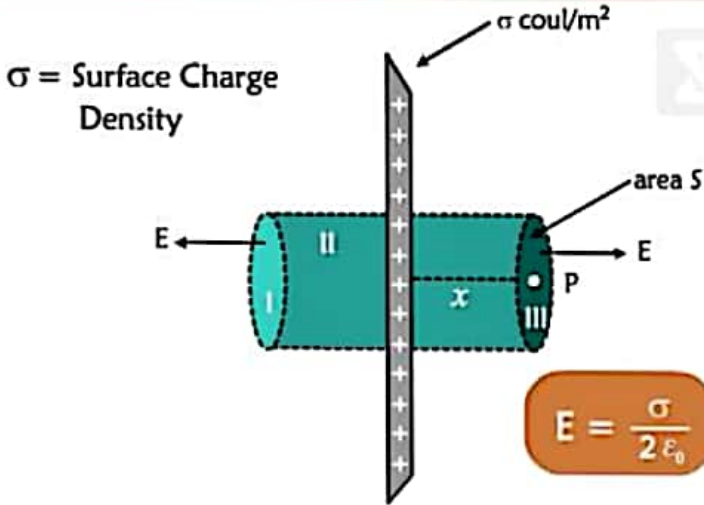
$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

Electric Field Strength due to a Long Charged Wire



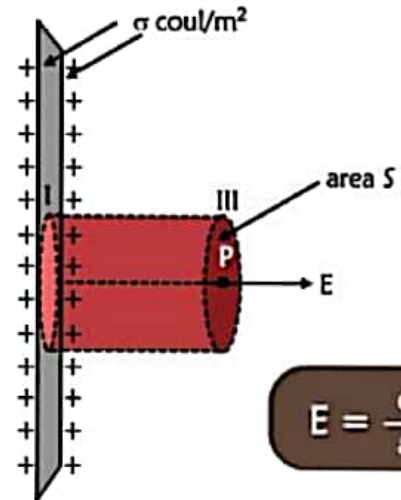
$$E = \frac{\lambda}{2\pi\epsilon_0 x}$$

Electric Field Strength due to Non-Conducting Uniformly Charged Sheet



$$E = \frac{\sigma}{2\epsilon_0}$$

Electric Field Strength due to Charged Conducting Sheet

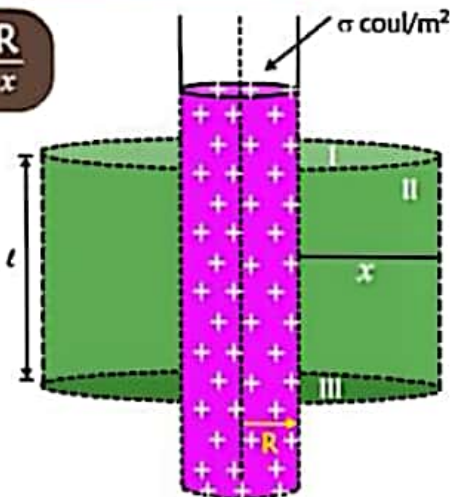


$$E = \frac{\sigma}{\epsilon_0}$$

Electric Field Strength due to a Long Uniformly Charged Cylinder

Conducting Cylinder

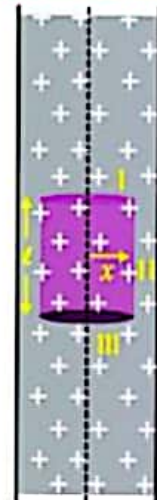
$$E = \frac{\sigma R}{\epsilon_0 x}$$



Uniformly Charged Non-Conducting Cylinder

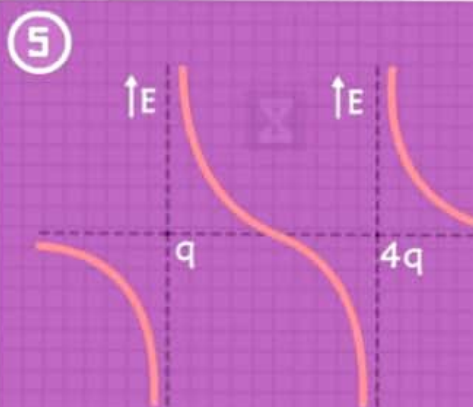
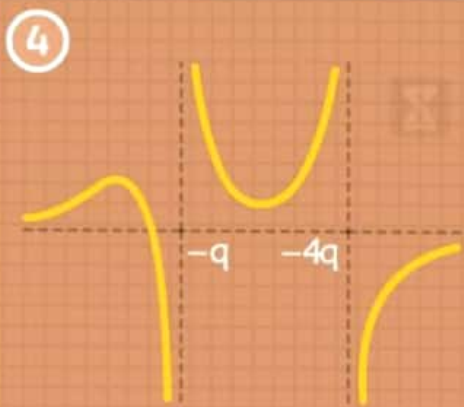
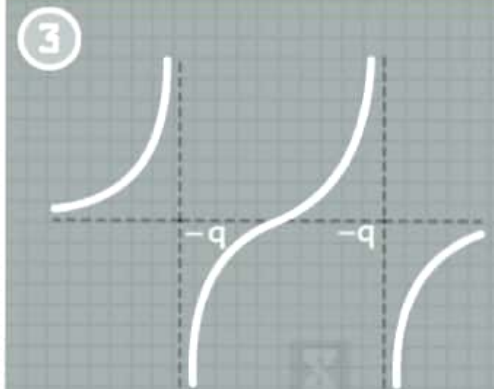
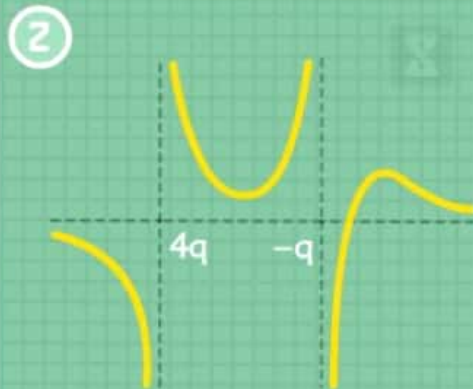
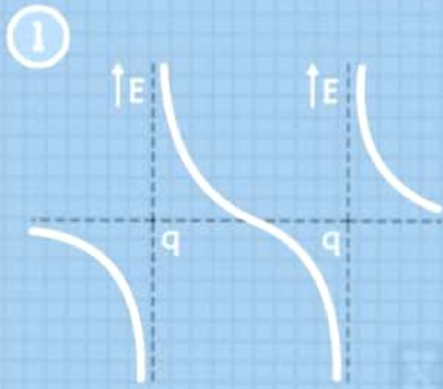
$$E_{\text{inside}} = \frac{\rho x}{2\epsilon_0}$$

ρ = Volume Charge Density



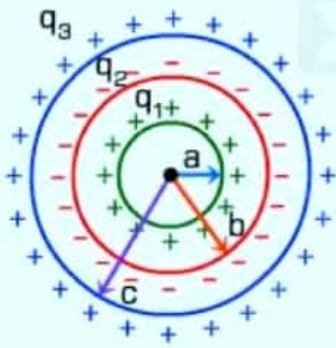
GRAPH OF ELECTRIC FIELD DUE TO BINARY CHARGES

Part IV



ELECTRIC POTENTIAL

POTENTIAL DUE TO CONCENTRIC SPHERES



At a point $r > c$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q_1 - q_2 + q_3}{r}$$

At a point $a < r < b$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r} - \frac{1}{4\pi\epsilon_0} \frac{q_2}{b} + \frac{1}{4\pi\epsilon_0} \frac{q_3}{c}$$

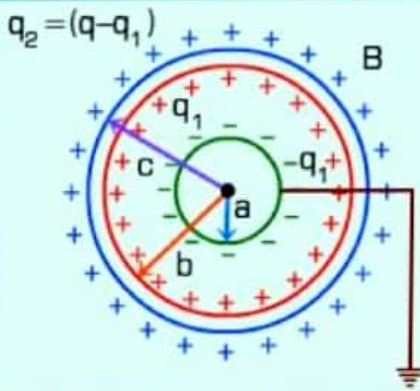
At a point $b < r < c$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q_1 - q_2}{r} + \frac{1}{4\pi\epsilon_0} \frac{q_3}{c}$$

At a point $r < a$

$$V = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{a} - \frac{q_2}{b} + \frac{q_3}{c} \right]$$

DIFFERENCE BETWEEN TWO CONCENTRIC SPHERES WHEN ONE OF THEM IS EARTHED



$$V_{in} = \frac{1}{4\pi\epsilon_0} \left[-\frac{q_1}{a} + \frac{q_2}{b} \right]$$

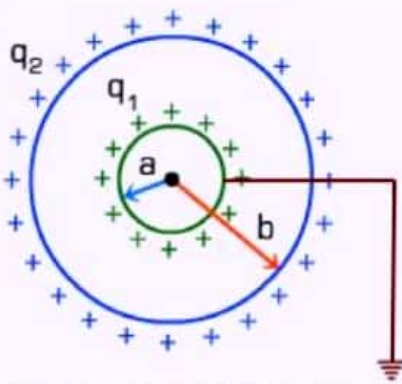
$$V_{out} = \frac{1}{4\pi\epsilon_0} \left[-\frac{q_1}{b} + \frac{q_2}{b} \right]$$

$$\frac{q_2}{c} = q_1 \left(\frac{1}{a} - \frac{1}{b} \right) \dots\dots(i)$$

$$q_1 + q_2 = q \dots\dots(ii)$$

Solving (i) and (ii) we can get q_1 and q_2

DIFFERENCE BETWEEN TWO CONCENTRIC UNIFORMLY CHARGED METALLIC SPHERES

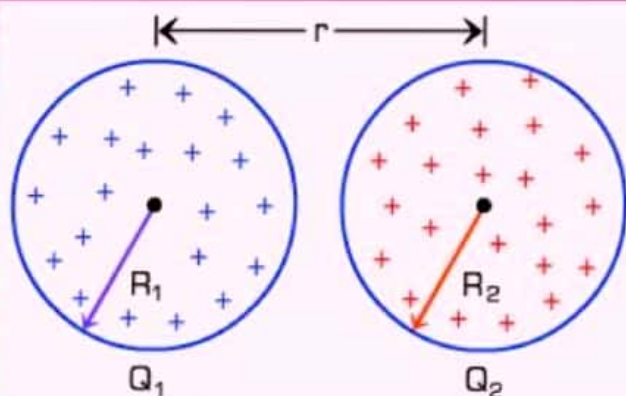


$$V_{in} = \frac{1}{4\pi\epsilon_0} \frac{q_1}{a} + \frac{1}{4\pi\epsilon_0} \frac{q_2}{b}$$

$$V_{out} = \frac{1}{4\pi\epsilon_0} \frac{q_1}{b} + \frac{1}{4\pi\epsilon_0} \frac{q_2}{b}$$

$$\Delta V = V_{in} - V_{out} \Rightarrow \Delta V = \frac{q_1}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right)$$

TOTAL ELECTROSTATIC ENERGY OF A SYSTEM OF CHARGES



$$U = U_{self} + U_{interaction}$$

$$U = \frac{3KQ_1^2}{5R_1} + \frac{3KQ_2^2}{5R_2} + \frac{KQ_1Q_2}{r}$$

ELECTRIC DIPOLE

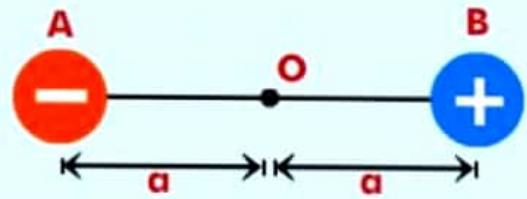
ELECTRIC DIPOLE

$$\vec{p} = q \cdot 2\vec{a}$$

SI unit : Coulomb - meter

It is a vector quantity

Direction of dipole moments (\vec{p}) is from negative charge to positive charge



ELECTRIC FIELD ON AXIAL LINE OF AN ELECTRIC DIPOLE

For $a \ll r$

$$E = \frac{1}{4\pi\epsilon_0} \frac{2qa}{(r^2 - a^2)^2}$$

$$\vec{E}_{\text{axial}} = \frac{1}{4\pi\epsilon_0} \frac{2\vec{p}}{r^3}$$

E_{axial} is along the direction of dipole moment

ELECTRIC FIELD ON EQUATORIAL LINE OF AN ELECTRIC DIPOLE

For $a \ll r$

$$E = -\frac{1}{4\pi\epsilon_0} \cdot \frac{q \cdot 2a}{(r^2 - a^2)^{3/2}}$$

$$\vec{E}_{\text{equatorial}} = -\frac{1}{4\pi\epsilon_0} \cdot \frac{\vec{p}}{r^3}$$

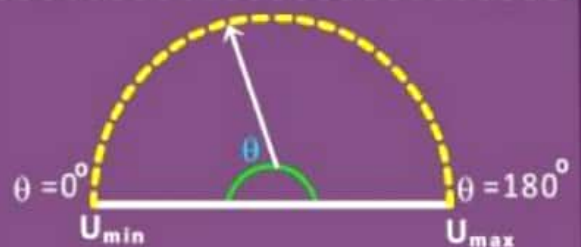
$E_{\text{equatorial}}$ is along the opposite direction of dipole moment

DIPOLE IN A UNIFORM EXTERNAL ELECTRIC FIELD

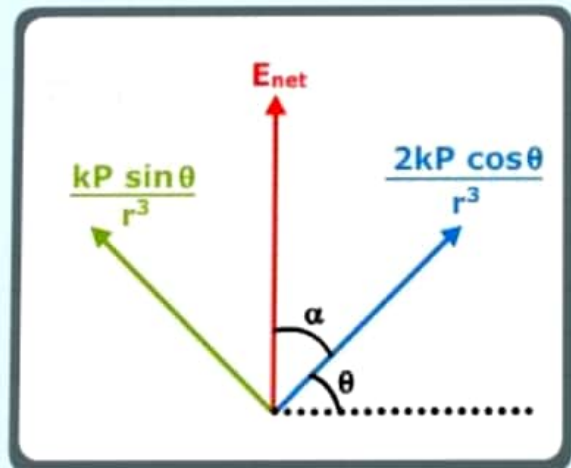
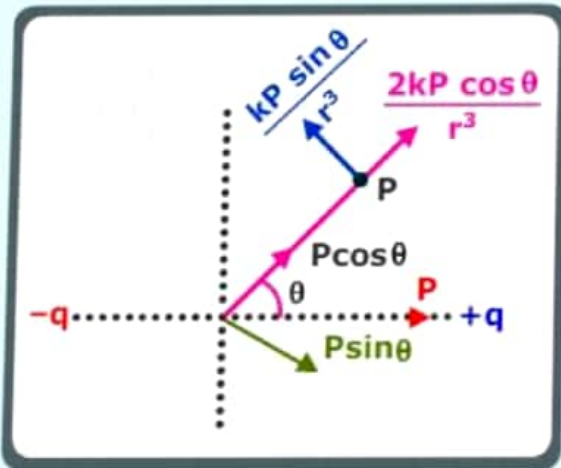
VECTOR FORM

$$\vec{\tau} = \vec{p} \cdot \vec{E}$$

$$U = -pE \cos \theta$$

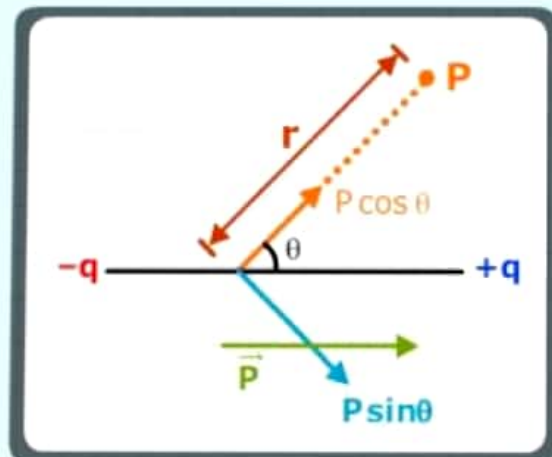
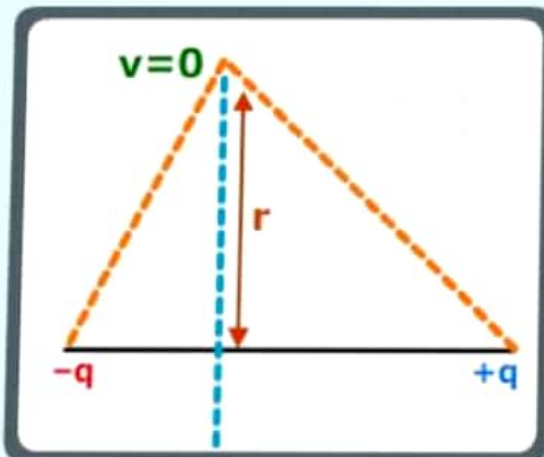


ELECTRIC FIELD AT A GENERAL POINT DUE TO A DIPOLE



$$E_{\text{net}} = \frac{kP}{r^3} \sqrt{1 + 3\cos^2 \theta}, \quad \tan \alpha = \frac{\tan \theta}{2}; \quad k = \frac{1}{4\pi\epsilon_0}$$

ELECTRIC POTENTIAL DUE TO A DIPOLE



POTENTIAL AT 'P' DUE TO DIPOLE, $V_p = \frac{2kP \cos \theta}{r^2}$

AT AN AXIAL POINT, $V_{\text{net}} = \frac{kp}{r^2}$ (As $P = q \cdot 2a$)

AT PERPENDICULAR BI-SECTOR, $V_{\text{net}} = 0$